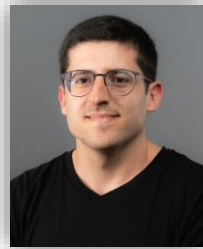




# Alias-Free Convnets: Fractional Shift Invariance via Polynomial Activations



Hagay Michaeli



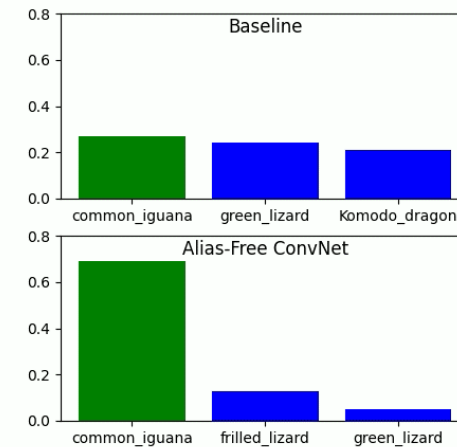
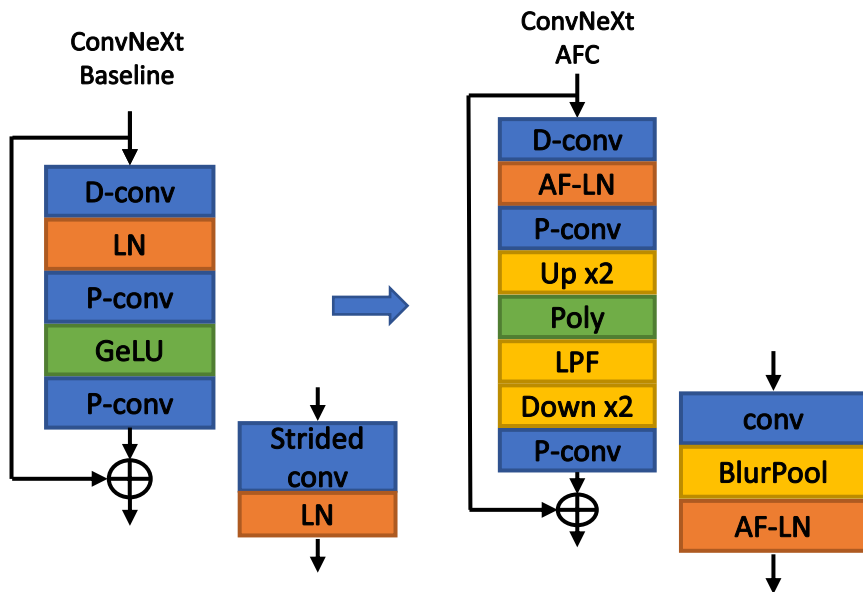
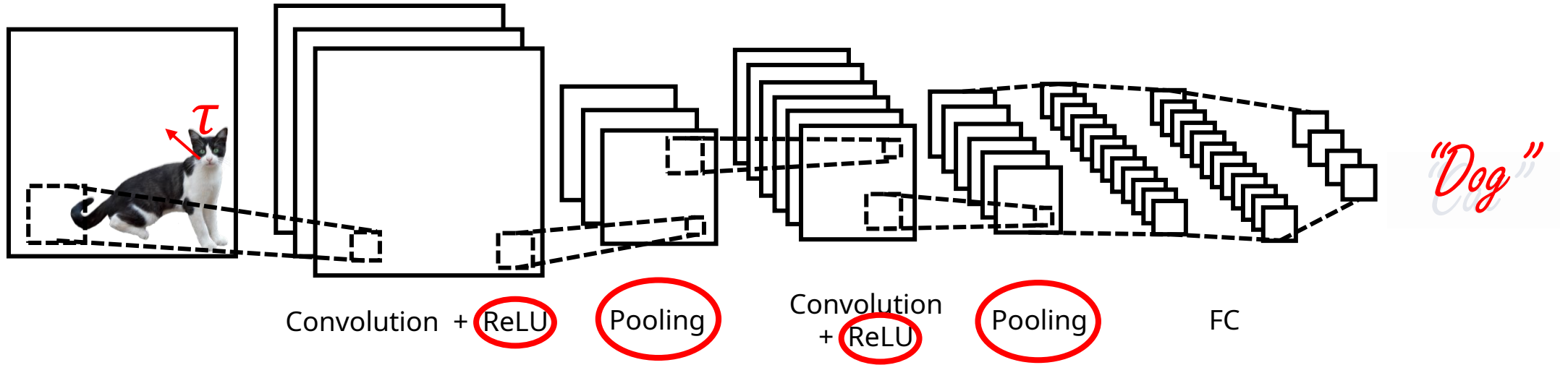
Tomer Michaeli



Daniel Soudry

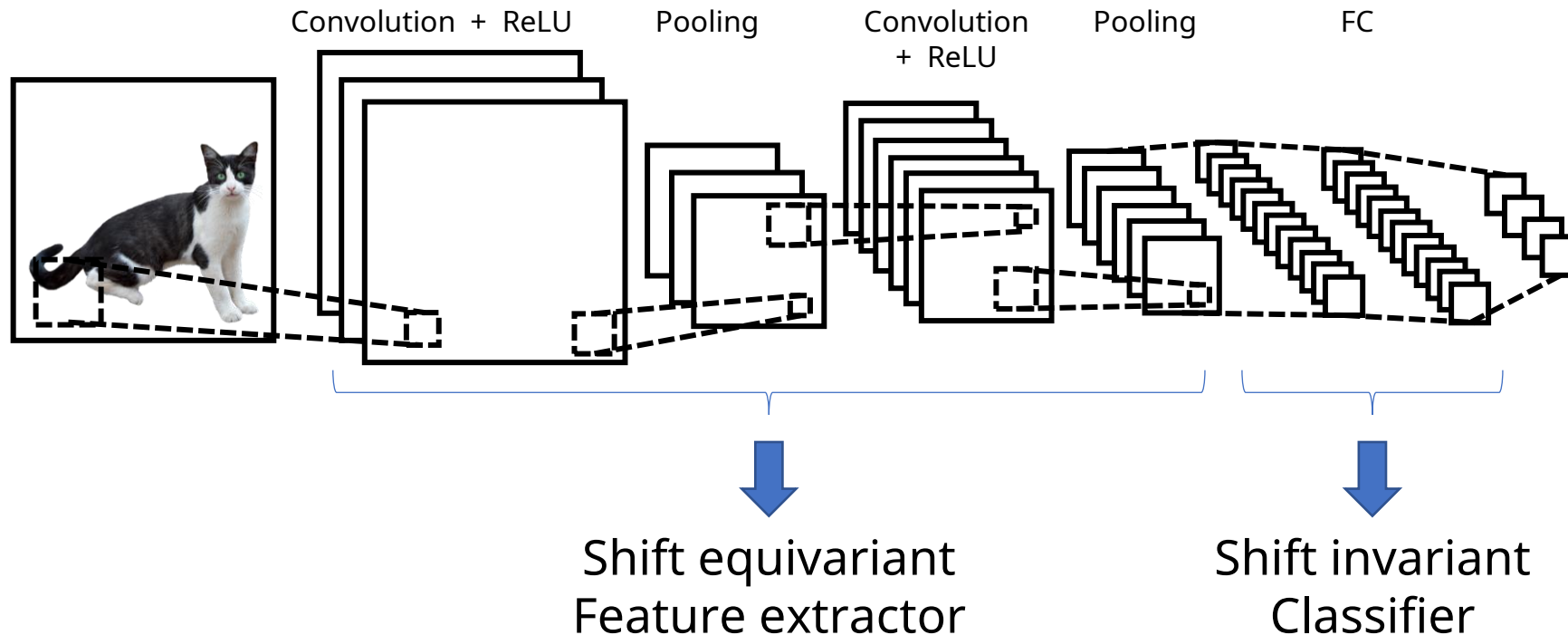
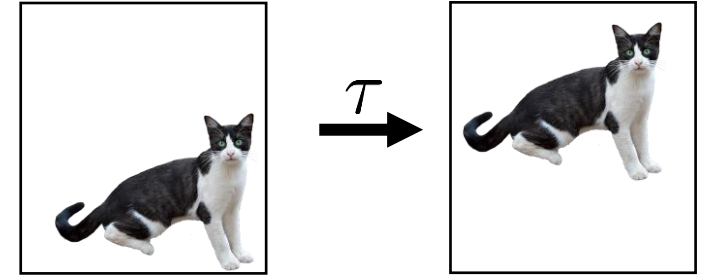


# Abstract

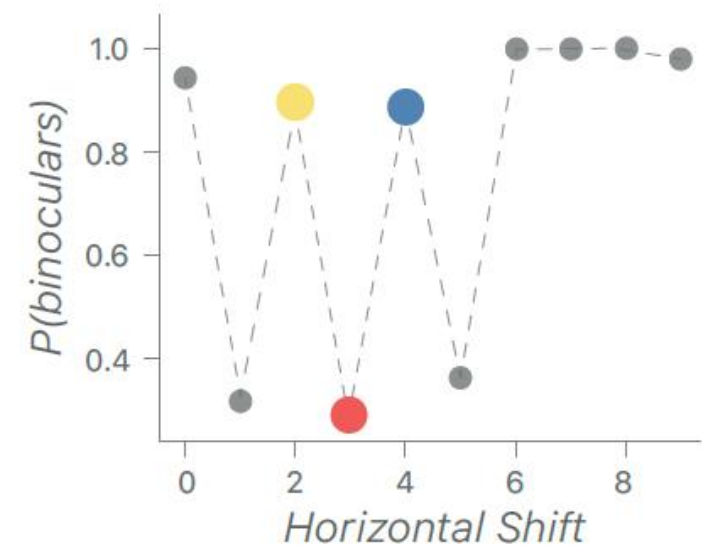
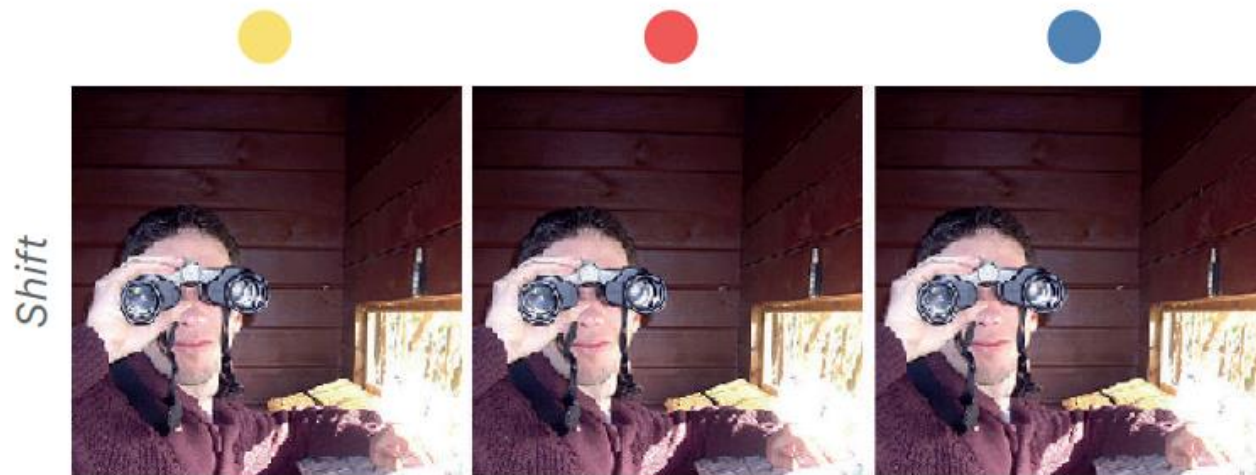


# Convnets: built-in priors

- Label unaffected by image shift
- Classifier should be “shift invariant”
- Features should shift with image
- Convnet hidden layers should be “shift equivariant”



# Single pixel shifts can change classification



**Why do deep convolutional networks generalize so poorly to small image transformations?**

Azulay & Weiss, JMLR 2019

# Is Feature extractor equivariant?

## Building blocks:

- Convolution layers
- Activation Layers (e.g., ReLU, GeLU)
- Normalization layers
  
- Pooling layers

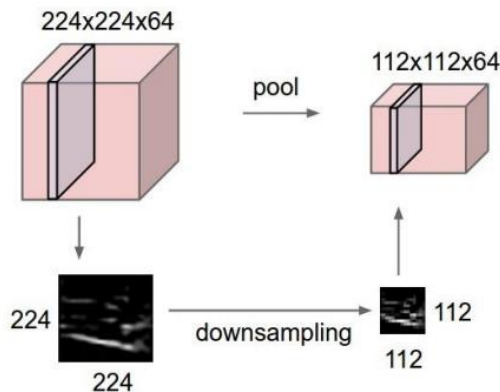
## Equivariant?

Yes

Yes

Yes (for some types)

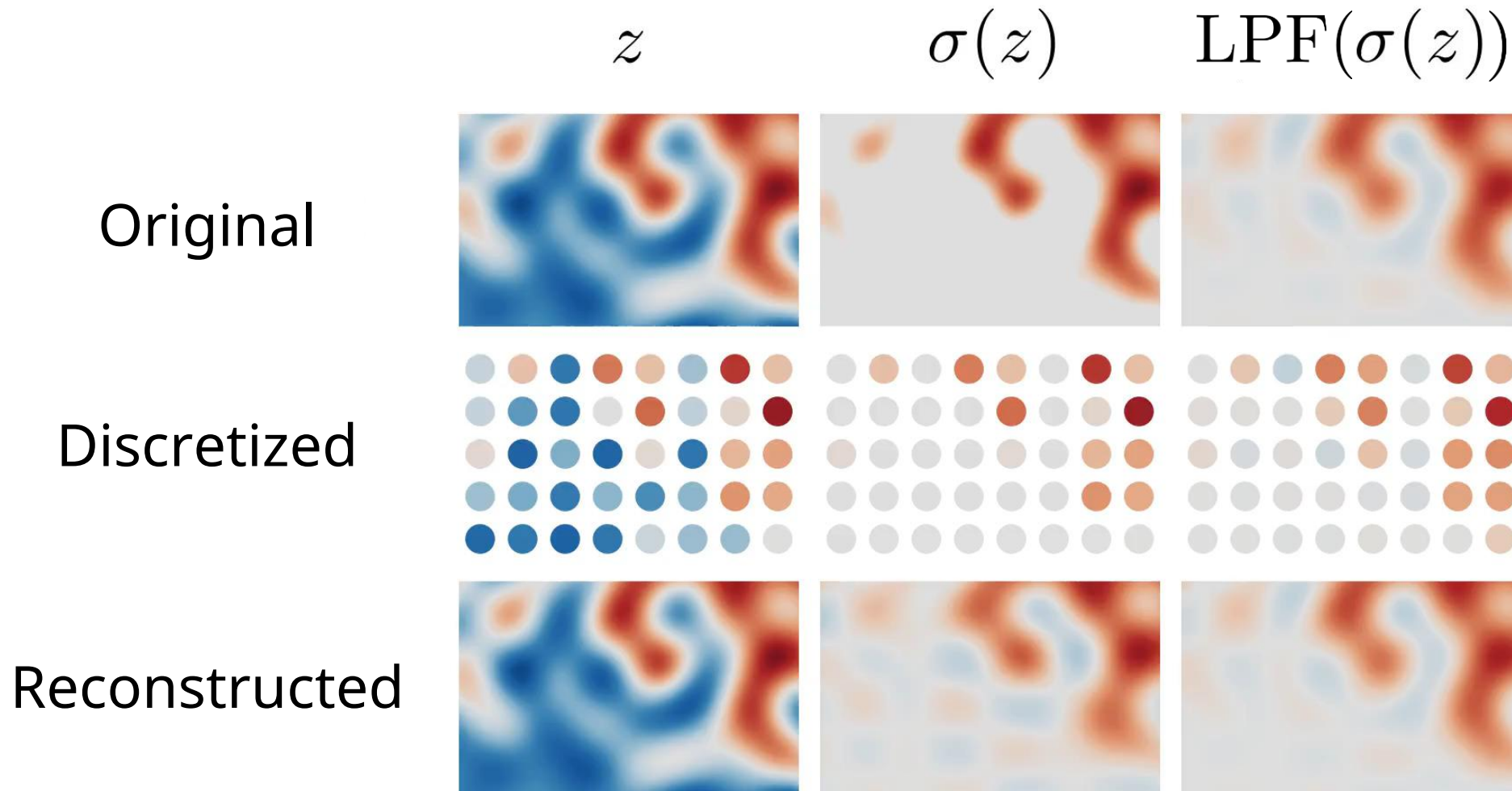
No



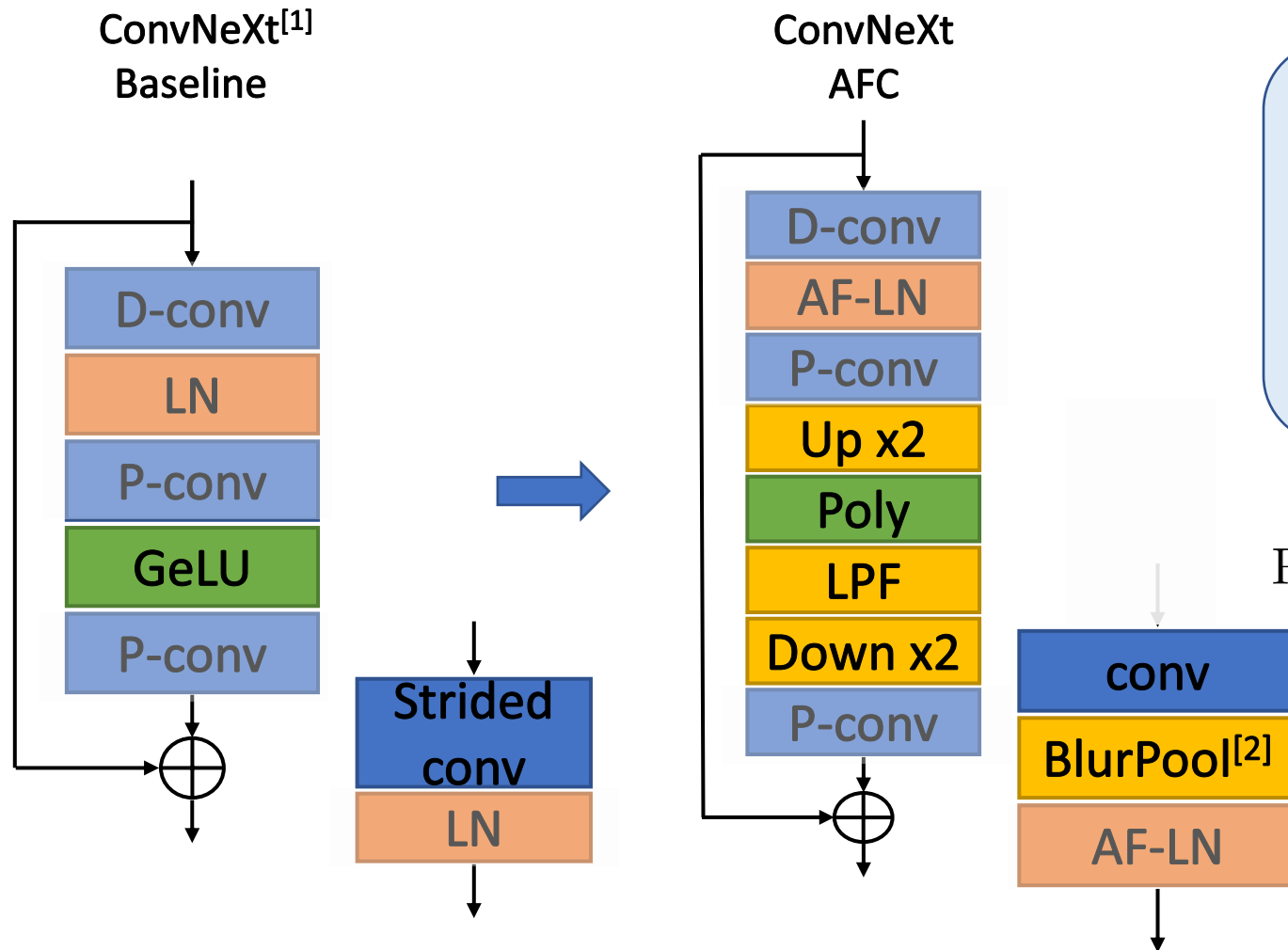
# What is the problem? Aliasing!

**Nyquist Theorem:** Perfect reconstruction if sampling rate  $> 2 \times$  bandwidth

# What is the problem? Aliasing!



# Alias Free Convnet (AFC)

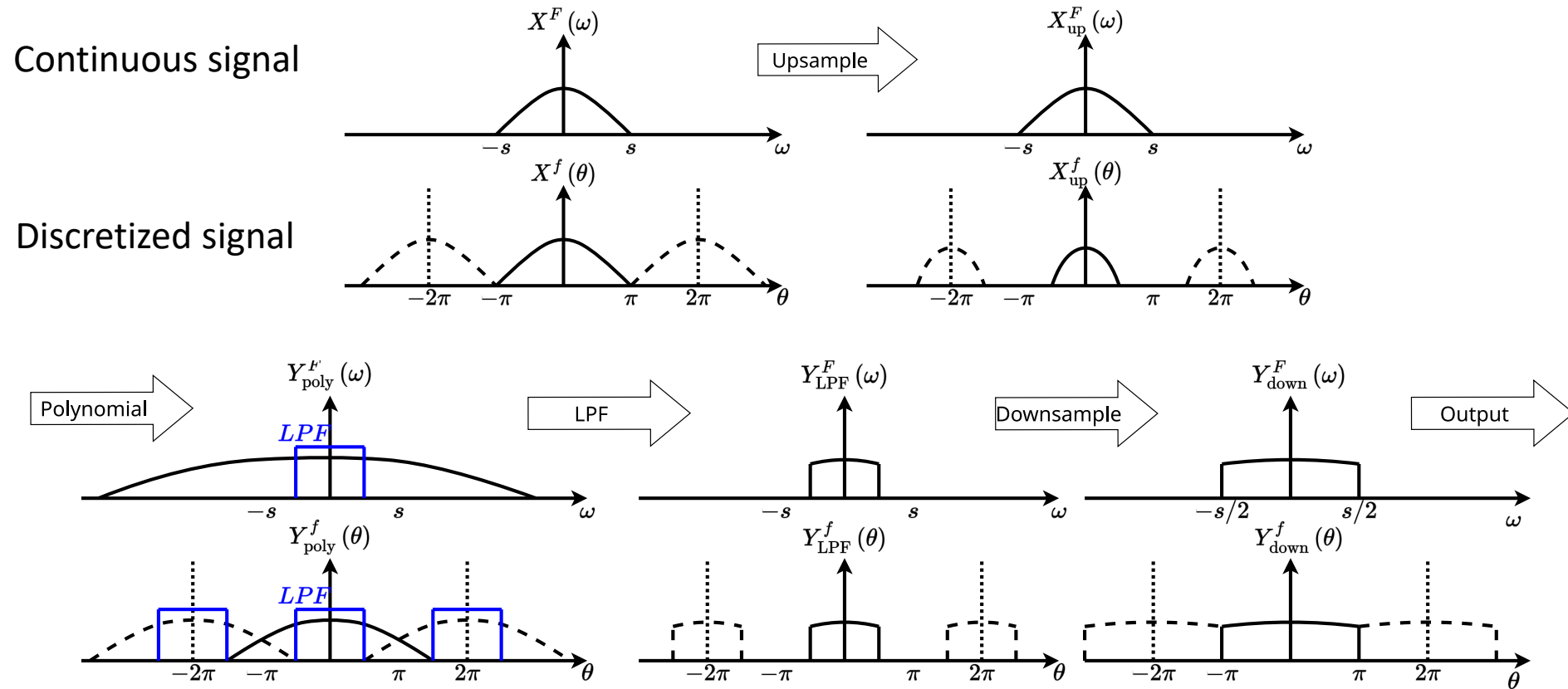


**Theorem:** AFC have perfect  
1) feature equivariance  
2) classifier invariance

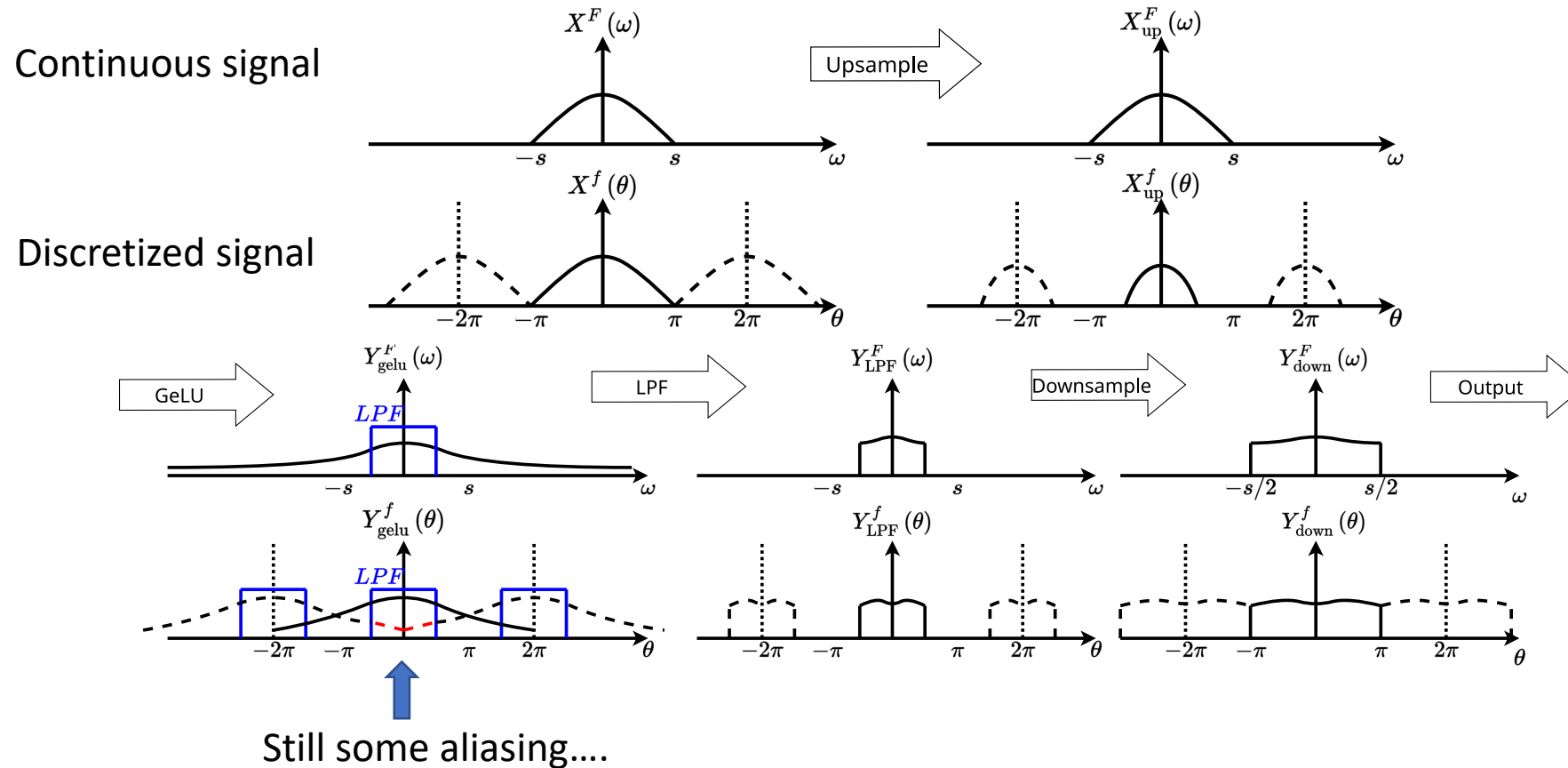
$$Poly(x) = a_0 + a_1x + a_2x^2$$



# Why this works? A look in the frequency domain

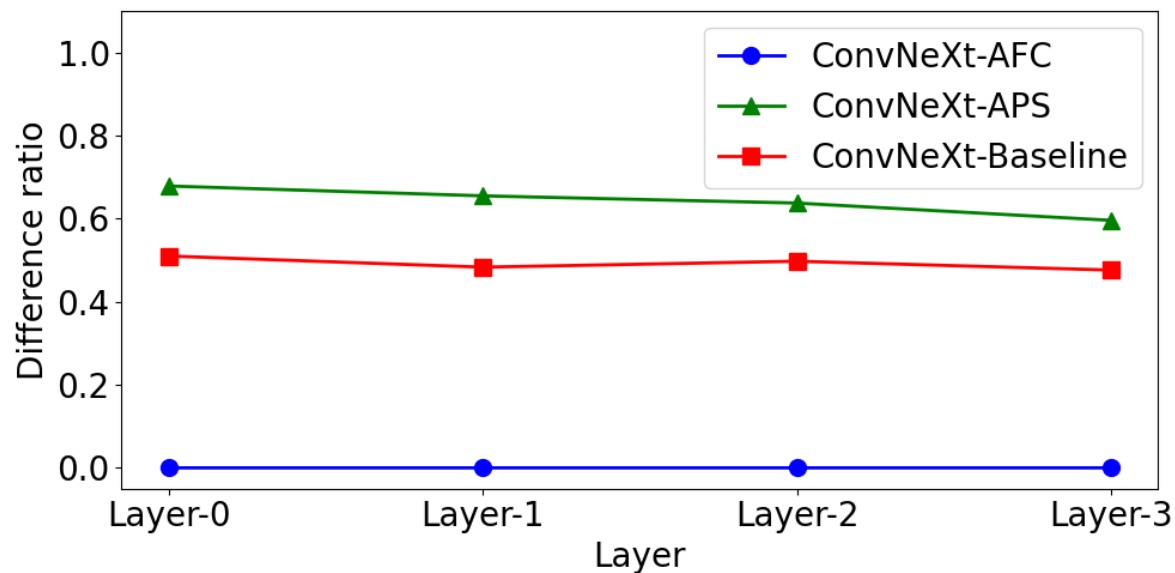


# What's the problem without polynomials?



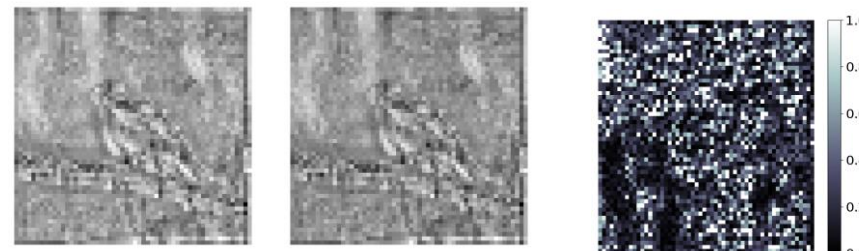
# Results (1): Perfect Equivariance

- Representations change after ½ pixel shift:

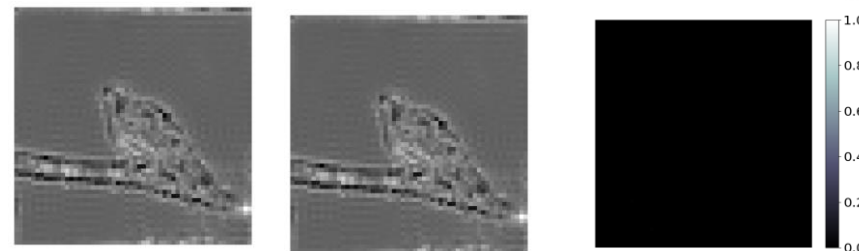


$$\left( \text{diff} \triangleq \frac{1}{CHW} \sum_{c,i,j} \frac{|y_{c,i,j}^0 - y_{c,i,j}^1|}{\max(|y_{c,i,j}^0|, |y_{c,i,j}^1|) + \varepsilon} \right)$$

APS<sup>1</sup>



AFC

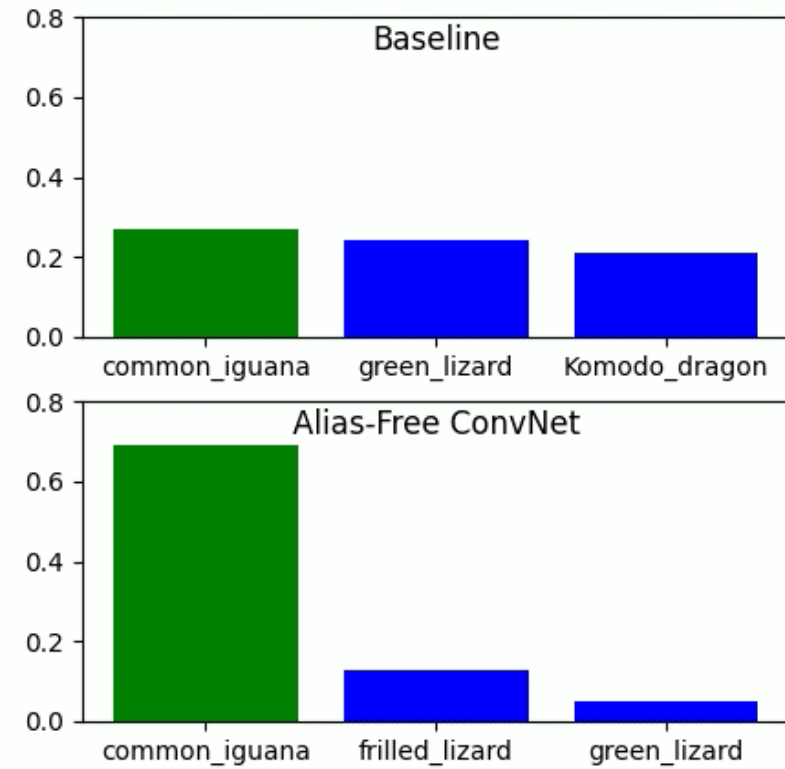


Original

½ pixel shift

Difference ratio

# Results (2): Perfect Invariance



# Results (3): Certified adversarial accuracy

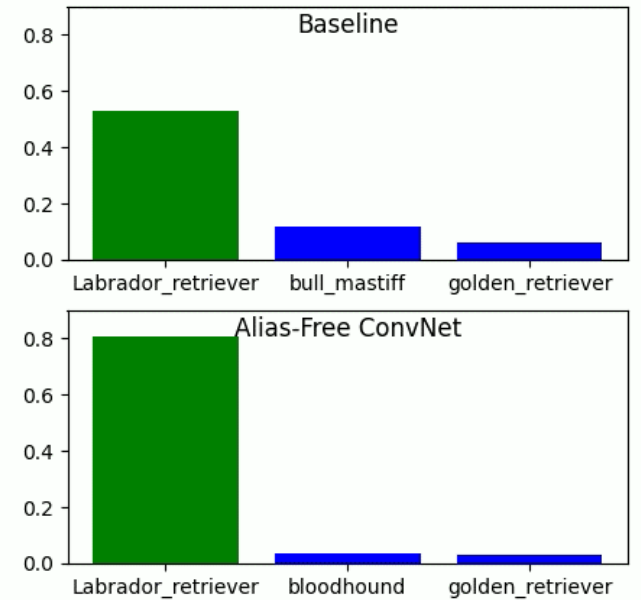
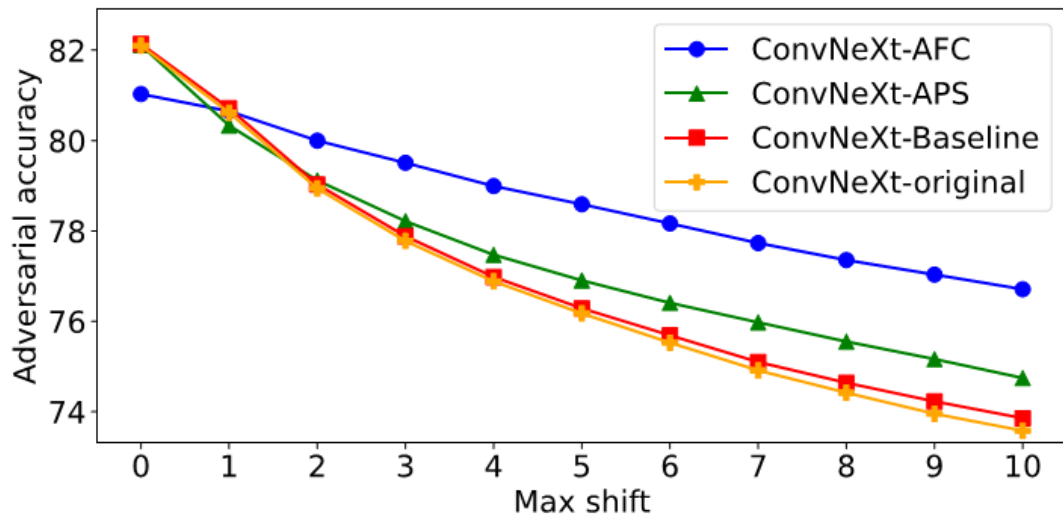
Model (ImageNet)	Test Accuracy	Integer grid	Half-pixel grid	Fractional grid
ConvNeXt-Baseline	<b>82.12</b>	76.63	73.65	77.82
ConvNeXt-APS	82.11	<b>82.11</b>	79.68	76.31
ConvNeXT-AFC (ours)	81.04	81.04	<b>81.04</b>	<b>81.04</b>

$$T_{\text{integer}} = \{(i, j) \mid 1 \leq i, j \leq 31\}$$

$$T_{\text{half}} = \left\{ \left( \frac{i}{2}, \frac{j}{2} \right) \mid 1 \leq i, j \leq 63 \right\}$$

$$T_{\text{frac}} = \left\{ \left( \frac{m_1}{n_1}, \frac{m_2}{n_2} \right) \mid 1 \leq m_{1,2} \leq n_{1,2} \leq 12 \right\}$$

# Results (4): Improved robustness to “camera translations”



# Summary: “Alias free convnets”

- Perfect invariance and equivariance to fractional shifts
- SOTA accuracy with adversarial fractional shifts
- First polynomial network with near-SOTA on ImageNet

Thank you!